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Water Allocation for Economic Production in a Semi-arid Region

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ABSTRACT *As water demand surpasses water availability, the problem of who will have access to water and who will be rationed is inevitable. This is already the case in arid regions or where the economic uses of water exceed water capability. This work contributes to the understanding and resolution of this decision-making process. Several allocation mechanisms are discussed and an allocation model based on the opportunity cost of water for different users is presented. This model leads to both a water pricing scheme and a method for compensating rationed parties.*

Introduction

During the 20th century, the world population was multiplied by 3 and the use of water was multiplied by 6 (Szöllözi-Nagy, 2000). The topic of 'water allocation' has been the subject of several studies, especially in recent years, since water use conflicts are becoming increasingly frequent, in several parts of the world. A recent compilation available from the Agricultural Research Service, an agency connected to the US Department of Agriculture (www.nal.usda.gov/wqic/Bibliographies/drought.hrm), lists 71 studies performed between 1992 and 1999, on droughts and water allocation. Among the most frequently studied topics are: (1) allocation criteria (Ridgley, 1993; Burton, 1994; Dinar *et al.*, 1999); (2) river basin management models (Chapman *et al.*, 1995; Hannan & Coals, 1995); and (3) legal and institutional aspects (Rosegrant & Schleyer, 1996; Llanos & Bos, 1997; Svendsen & Meinzen-Dick, 1997; Asad *et al.*, 1999).

Ideally, the allocation of water should be economically efficient and socially fair. However, these objectives are frequently in conflict. An economically efficient allocation seeks to distribute water maximizing the added economic benefit produced in the river basin, without distinguishing who are the beneficiaries. Allocation with social equity, on the other hand, seeks to distribute water in an attempt to protect the users' interests with less participation in the added economic benefit.

There is a growing understanding that the integrated management of water resources includes treating water as an economic asset. However, there is little agreement about what exactly this means (Briscoe, 1996, 1997; Dinar, 1997). Recently, a number of studies have compared advantages and disadvantages in water markets, which seek economic efficacy, with different alternatives for the

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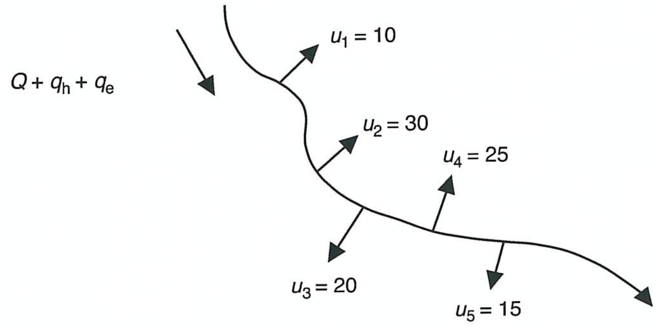

Figure 1. Five users (all consumptive) in a river reach without a reservoir.

state administration of water rights that generally try to achieve social equity. This study presents a proposal for a methodology to ration water that at the same time tries: (1) to maximize the added value of production depending on water as an input, on the river basin scale; and (2) to compensate, financially, those who undergo rationing from a fund fed by contributions by those who do not undergo rationing.

In a situation in which there is rationing, it is preferable to make all users be given the same percentage of the potential net income than given the same percentage of the water demanded. In the example in this paper, it is shown that with 80% water availability in relation to the added demand of all users, it is possible to produce a liquid benefit of 90% for each user, in relation to the potential benefit, in a situation of non-rationing. A didactic approach is adopted using simple numerical examples.

Consumptive Use

Figure 1 shows the situation in which five users utilize water as the input to an economic process in a same reach of river or canal where there is no reservoir with a significant regulation capacity. If u_1 is the water demand of user 1, located at the most upstream point in the reach, during the time interval considered, analogously, $u_2, u_3, \ldots, u_5$ represent the demands of the other users, arranged from upstream to downstream. It is observed that the total demand in the time interval considered is $d = \Sigma u_i = 100$ volume units. In this example, $1000\ \mathrm{m}^3$ is adopted as a volume unit. The time interval could be a year, month or day, and will be generically called Δt. For simplicity's sake, let us accept that there is no return flow. In other words, we will accept that the volume taken up by each user is equal to the volume consumed.

Let $Q + q_h + q_e$ be the volume of water flowing into the reach immediately upstream from user 1 during time interval Δt. Q is called 'water availability', for production purposes, q_h is called a 'water reserve for human consumption and watering the animals' in that reach,[1] and in those located downstream, q_e is called a 'water reserve to preserve the river ecosystem'. This form of dividing the inflowing volume is particularly appropriate for countries such as Brazil, in which the law prioritizes water use for these 'non-economic purposes'. However, the formula is also valid for other countries that do not have this legal disposition, as is the case for the USA, and it is sufficient to assume that both q_h

and q_e are nil. In fact, throughout the rest of this text the authors will assume, for simplicity's sake, that the inflowing volume is always larger than $q_h + q_e$. In other words, they will assume that there will be no lack of water to fulfil the basic non-economic needs. Thus, this paper treats water distribution as an input for the production process.

Naturally, Q is a random variable. If water availability is 'never' less than the demand, i.e. if $P(Q < d) = 0$, there is no allocation problem.[2] This is a situation in which the most appropriate legal model follows the 'riparian' doctrine: whoever owns riparian lands has the right of access to water. On the other hand, if $P(Q < d) > 0$, i.e. if the probability of rationing is not negligible, there will be situations in which one or more users cannot be entirely satisfied. In the absence of any rationing system, there is no way to reserve water for the priority uses, including human consumption. The natural order of access priority will be from upstream to downstream, i.e. user 2 can only receive what they need after user 1 has withdrawn from the river or canal all of their consumption, u_1. Analogously, user 3 can only be attended to after users 1 and 2 have withdrawn all their consumptions $u_1 + u_2$ from the river or canal; and so on, successively. In other words, the probability of rationing user 1 is $P(Q < u_1)$, of user 2 it is $P(Q < u_1 + u_2)$, of user 5 it is $P(Q < u_1 + u_2 + u_3 + u_4 + u_5)$. Clearly, in the absence of tributaries in the stretch considered, as, for instance, in the case of the example, the probability of rationing grows from upstream to downstream. This rationing system obeys the 'law of the water jungle', enunciated as follows: he who is upstream is allowed anything; whoever is downstream better get used to it. This is a system without any foundation, whether it be legal, economic or social. It is precisely the situation now observed in many regions where water is scarce in developing countries, including the semi-arid northeast of Brazil.

In basins where there are water scarcity problems, it is useful to establish a system of priority of access based on some rationality. Next the authors will look at several alternatives. All of them require that a system of water use rights be implemented in the river basin, and that the corresponding control mechanisms be installed. The resulting cost should be shared among all users. The most reasonable solution would be to charge a contribution proportional to demand from each user. In other words, each user i should contribute a part equal to βu_i, where β is the unit price of the water use right, for purposes of maintenance of the administrative and control system.

Usually, the first idea that arises, when alternatives are discussed for the reduction of water use in situations of scarcity, is to assume the linear rationing system. In this alternative, each user would have the effective use of water proportional to the requested demand and is subject to a probability of rationing identical to that of the other users. For instance, if $Q = 80$, each user's consumption would be reduced by 20%. This is a rationing system that is legally rational and easy to enunciate and understand. However, it is difficult to implement, because it depends on a very sophisticated methodology for control, i.e. the value of β would be very high.

In the western USA, since the 19th century a system of rationing with low administrative costs (small β) based on the 'appropriation doctrine' or 'chronological doctrine' has been adopted. In this system, the priority of use, in a case of rationing, is greater for those who have been using water for a longer time, according to a grant given by the respective state government. Everything is done as though the users were in a queue. Any occupant of the queue

Table 1. Two systems to define priorities

User i	Demand u_i (1000 m^3)	Date of grant	Priority of access to water: 'Law of the jungle'	Priority of access to water: 'Chronological'
1	10	1961	1	3
2	30	1932	2	1
3	20	1975	3	4
4	25	1980	4	5
5	15	1955	5	2
	$d = 100$			

has priority over the other users behind him. This procedure is simple to control by the users themselves, which explains the low value for β. The grant is characterized by demand and the recognized date when operations began.[3] In recent years, grants have been sold on 'water markets'. In most rivers, the state governments no longer give new grants, because the risk of rationing the user with a more 'junior' right (the last place in the queue) would be higher than tolerable for any economic activity. Therefore, in the western USA, the binomial of the right to use a quantity of water and its corresponding priority is private property, without any link to land ownership.

Let us assume that the date when water began to be utilized by users in Figure 1 is given by the third column in Table 1 (for simplicity, only the year when use began is shown). The fifth column indicates what the order of priority would be according to this rationing system.

This is a rationing system that has a legal rationale and is simple to implement. For instance, if, at the water intake of user 5, 15 or more volume units are not passing through in the period considered (its demand), and if any of the users with a lower priority (users 1, 3 and 4) are using water freely, all user 5 has to do is call in the agent responsible for control, the 'water commissioner', to impose his rights. The water commissioner has the prerogative of switching on the hydraulic structures that control the water flows.

As in the case of the rationing system, based on the 'law of the jungle', the adoption of the chronological method also results in a different probability of rationing for each user. In the example, the probability of rationing for user 1 is $P(Q < u_2 + u_5 + u_1)$, for user 2 it is $P(Q < u_2)$, of user 5 it is $P(Q < u_2 + u_5)$. Clearly, the probability of rationing is higher for users with more recent water use grant rights.

In countries where no register for water use grants has been established, an initial allocation of the right to use and respective priorities could be carried out by means of an auction. For instance, the granting powers could limit the total requested demand d in such a way that the probability of some rationing would be, for instance, 5%. In other words, the total demand d would be chosen such that $P(Q < d) = 0.05$. This total demand would then be divided into k lots, each corresponding to d/k volume units, with a priority ranging from 1 to k. Naturally, the lots with a higher priority would be auctioned for values higher than those of the lots with a lower priority. The risk of this alternative is that some users in the more highly informed sectors will end up taking over water

Table 2. Three systems to define priorities

User i	Demand u_i (1000m^3)	Date of grant	Unit benefit b_i (\$/1000 m^3)	Priority of water access: 'Law of the jungle'	'Chronological'	'Benefit'
1	10	1961	5	1	3	4
2	30	1932	7	2	1	3
3	20	1975	10	3	4	1
4	25	1980	8	4	5	2
5	15	1955	3	5	2	5
	$d = 100$					

lots superior to the real needs, simply for long-term speculation. That is what was done in the electricity sector of Chile during the 1990s.[4]

An alternative, to preserve the simplicity of the rationing method adopted in the west of the USA—the queuing method—and avoid the risk of the single auction, would be to hold annual auctions, organized by the granting body, in order to give a one-year concession for the use of water as an input to the production process, once the part reserved for human and animal consumption is discounted.

The auction can be substituted, advantageously, by a system in which each user declares their demand and their unit net benefit (for instance in dollars per 100 m^3) to the granting authority. The unit net benefit is the profit that the user estimates that he had due to the use of a volume unit of water during the time interval taken into account, assuming that this water has no cost, and having discounted all the other costs (including β). The priority of access to water is established in the inverse order of this unit benefit. Continuing the previous example, the fourth column of Table 2 shows the unit benefits, while the seventh column shows the corresponding priorities according to this system, henceforth called 'benefit'.

The unit benefit b_i is the highest price of water that the user i would be prepared to pay in order not to be rationed. This is the unit price that will only be practised in a situation of rationing, besides the unit price for maintenance and control β. If the unit price of water rationing p is higher than the unit benefit, it would be preferable to have rationing. For instance, if $p = \$6/1000\ \text{m}^3$, user 1 prefers to be rationed and not profit \$5/1000 m^3 than to pay the \$6/1000 m^3 in order not to undergo rationing. On the other hand, user 2 prefers to pay the \$6/1000 m^3 not to be rationed, than not profit \$7/1000 m^3.

In the 'benefit' rationing method, the probability of rationing for user 1 is $P(Q < u_3 + u_4 + u_2 + u_1)$, for user 2 it is $P(Q < u_3 + u_4 + u_2)$, for user 5 it is $P(Q < u_3 + u_4 + u_2 + u_1 + u_5)$. Clearly, the probability of rationing is higher for users who obtain a lower economic benefit using the water.

The price of water for all unrationed users is equal to the unit benefit of the user who suffers partial rationing, called the 'marginal user'. This is the last user in the queue organized according to priorities established in the order of unit benefits. The other users, with a lower priority of access than the marginal user, undergo rationing and do not pay for anything.

Figure 2 illustrates the concept for the case in which water availability is

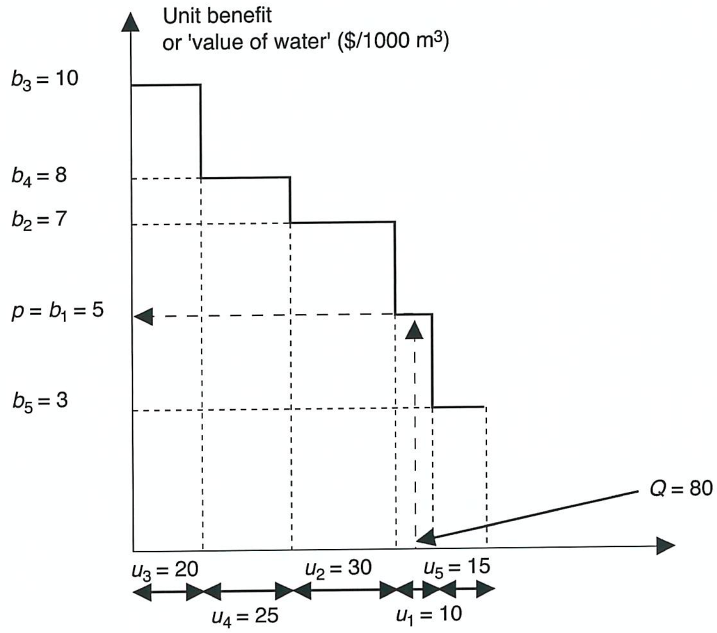

Figure 2. Determination of price of water under rationing.

$Q = 80$. In this example it is observed that the threshold user is user 1, resulting in a unit price of water in a rationing regime for all unrationed users equal to $p = b_1 = \$5/1000\ m^3$. Users 3, 4 and 2 do not suffer any rationing. User 1 suffers partial rationing and user 5 suffers complete rationing. All the unrationed ones pay $5/1000 m^3. As was to be expected the unit price of water in a rationing regime varies inversely with water availability. At the limit, when $Q > d$ this price is nil.

Figure 2 also can be seen as the graph that represents the variation of the value of water with water availability. 'Value of water' is the greatest unit benefit that can be produced if the water availability increases by one volume unit (for instance, 1000 m^3).

For demonstration purposes, Tables 3–6 show the allocation of water among

Table 3. The 'law of the jungle'

User i	Priority	Unit benefit b_i ($/1000 m^3)	Demand u_i (1000 m^3)	Volume provided q_i (1000 m^3)	Economic benefit b_iq_i ($)	Cost of water pq_i ($)	Results (benefit minus cost) ($)
1	1	5	10	10	50	0	50
2	2	7	30	30	210	0	210
3	3	10	20	20	200	0	200
4	4	8	25	20	160	0	160
5	5	3	15	0	0	0	0
Total			$d = 100$	$Q = 80$	620	0	620

Table 4. 'Linear'

User i	Priority	Unit benefit b_i ($/1000 m^3)	Demand u_i (1000 m^3)	Volume provided q_i (1000 m^3)	Economic benefit b_iq_i ($)	Cost of water pq_i ($)	Results (benefit minus cost) ($)
1	—	5	10	8	40	0	40
2	—	7	30	24	168	0	168
3	—	10	20	16	160	0	160
4	—	8	25	20	160	0	160
5	—	3	15	12	36	0	36
Total			$d = 100$	$Q = 80$	564	0	564

Table 5. 'Chronological'

User i	Priority	Unit benefit b_i ($/1000 m^3)	Demand u_i (1000 m^3)	Volume provided q_i (1000 m^3)	Economic benefit b_iq_i ($)	Cost of water pq_i ($)	Results (benefit minus cost) ($)
1	3	5	10	10	50	0	50
2	1	7	30	30	210	0	210
3	4	10	20	20	200	0	200
4	5	8	25	5	40	0	40
5	2	3	15	15	45	0	45
Total			$d = 100$	$Q = 80$	545	0	545

users, and the economic results, for each of the four rationing systems described, if $Q = 80$.

It is observed that the implementation of rationing based on the economic benefit results, as was to be expected, in the maximum total value for the economic benefit. However, part of this benefit is 'collectivized', by billing for water use in a situation of rationing, which is proportional to the volume provided. This billing, which only occurs during rationing, must not be confused with billing for water use for administrative and control purposes (unit price β), which occurs with or without rationing and is proportional to demand.

Table 6. 'Economic benefit'

User i	Priority	Unit benefit b_i ($/1000 m^3)	Demand u_i (1000 m^3)	Volume provided q_i (1000 m^3)	Economic benefit b_iq_i ($)	Cost of water pq_i ($)	Results (benefit minus cost) ($)
1	4	5	10	5	25	25	0
2	3	7	30	30	210	150	60
3	1	10	20	20	200	100	100
4	2	8	25	25	200	125	75
5	5	3	15	0	0	0	0
Total			$d = 100$	$Q = 80$	635	$pQ = 400$	235

Table 7. 'Economic benefit' with financial compensations

User i	Priority	Unit benefit b_i ($/1000 m^3)	Demand u_i (1000 m^3)	Volume provided q_i (1000 m^3)	Economic benefit b_iq_i ($)	Cost of water pq_i ($)	Financial compensation γ_i ($)	Result ($)
1	4	5	10	5	25	25	45	45
2	3	7	30	30	210	150	129	189
3	1	10	20	20	200	100	80	180
4	2	8	25	25	200	125	105	180
5	5	3	15	0	0	0	41	41
Total			$d = 100$	$Q = 80$	635	$pQ = 400$	400	635

The total amount collected by the managing body, when billing during a rationing situation, which, in the example, is equal to $400, can be applied to investments, funding activities or financial compensations that are in the collective interest of the users, according to a decision taken within the association of users or the river basin committee. Regarding the item 'financial compensation', particular attention must be given to rationed users, who are generally the economically most fragile. In developing countries, this compensation has the merit of avoiding even greater swelling of human agglomeration around large cities, basically formed by the remnants of former farmers' families. A reasonable criterion would be to distribute the compensation among all users, so that the result achieved by each user will constitute a fraction, equal for all, rationed and unrationed, of the respective maximum potential result b_iu_i. This maximum potential result is the value of the result that user i estimates that he will reach in a situation in which there is no rationing. If the total amount collected is returned to the users, according to this criterion, the compensation for user i should be equal to γ_i to be calculated according to the following set of equations:

$$\gamma_i + (b_i - p)q_i = \alpha b_i u_i \qquad \text{for all } i \tag{1}$$

The sum of the equations above for all users i results in:

$$pQ + \Sigma(b_i - p)q_i = \alpha \Sigma b_i u_i \tag{2}$$

Therefore:

$$\alpha = \Sigma b_i q_i / \Sigma b_i u_i \tag{3}$$

In the specific example, $\alpha = 0.9$. This means that the result for each user, in a situation in which there is rationing, is equal to 90% of what the result would be without rationing, much better than would be achieved using the linear method applied to volumes, whose result would be only 80%. Table 7 presents the economic results for the rationing system based on the economic benefit, already taking into account the financial compensations.

It is observed that the sum of results is the maximum, and that those who are undergoing rationing are not left out. On the contrary, they participate in the bonanza. Naturally, if a user is systematically rationed, year after year, something must be done so that he will not continue to benefit from wealth produced by the other users. In order to do so, it is sufficient that according to law any grant that goes unused, say for three successive years, lapses.

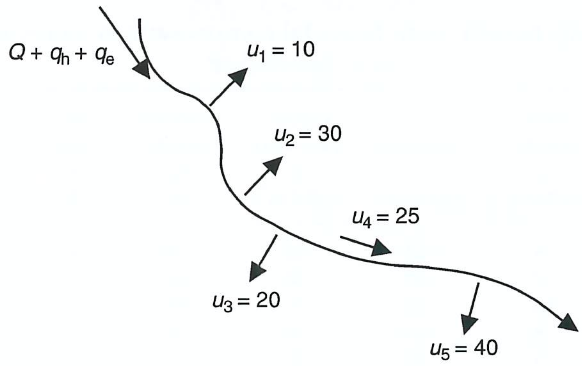

Figure 3. Five users (one non-consumptive) on a river reach without a reservoir.

Non-consumptive Use

Figure 3 shows the situation in which one of the users, number 4, does not consume water during the production process (that is why he is called a non-consumptive user), but who needs water in the river in order to achieve full production. This user could be, for instance, a run-of-the-river hydropower plant, a navigation company, an aquaculture business or a recreation firm. For the three last possibilities, it would be necessary to ensure a minimum depth and, consequently, minimum streamflow. Let us assume that the need of user 4 in interval Δt is equal to $u_4 + q_h + q_e$. It should be recalled that lot $q_h + q_e$ is already ensured by law for human supply, watering the animals and conservation of the river ecosystem. Therefore, it is in the interest of user 4 to ensure that $Q - q_1 - q_2 - q_3$ is larger than u_4. In order for the total consumptive demand to remain the same as in the previous example ($d = 100$), the demand of user 5 is increased from $u_5 = 15$ to $u_5 = 40$.

There are those who think that non-consumptive users do not have to enter the dispute for water when there is rationing, precisely because they do not consume water. The example that is being discussed shows that this idea is wrong. Suppose that user 4 is a hydropower plant that is 'hitching a ride', and decides not to participate in the collective effort for the optimum allocation of water. In this case user 4 is taken off the list, and users 1, 2, 3 and 5 are left on it. Consequently, all of the rationing is concentrated on user 5, who has the lowest priority. Therefore, the price of water is equal to $b_5 = \$3/1000\ \text{m}^3$. The volume provided for the users upstream from user 4 is, in this case, equal to $q_1 + q_2 + q_3 = 60$. Thus, in the time interval considered, only a volume equal to 80–60 = 20 volume units flows down the river passing in front of user 4. In this case, user 4 has a result equal to $b_4 q_4 = 8 \times 20 = \$160$. Table 8 shows the calculations made for this case. The line corresponding to user 4 is shown only for the sake of convenience. However, the totals of the last column are calculated ignoring the participation of user 4 in the basin. It is observed that the total result in the basin, including user 4, would be 160 + 520 = \$680.

Let us assume that user 4 decides to join the effort of the other users to allocate water in the collectively most efficient manner. In this case he has a common interest with the users that are downstream from him, as opposed to the users who are upstream from him. His best strategy is to establish a coalition with the

Table 8. 'Economic benefit' with financial compensations: non-consumptive user is a 'hitchhiker'

User i	Priority	Unit benefit b_i (\$/1000 m^3)	Demand u_i (1000 m^3)	Volume provided q_i (1000 m^3)	Economic benefit b_iq_i (\$)	Cost of water pq_i (\$)	Financial compensation γ_i (\$)	Result (\$)
1	3	5	10	10	50	30	25	45
2	2	7	30	30	210	90	68	188
3	1	10	20	20	200	60	39	179
4	—	8	(25)	(20)	160	—	—	160
5	4	3	40	20	60	60	108	108
Total			$d = 100$	$Q = 80$	520	$pQ = 240$	240	520

Note: 'Volume' means volume demand or volume provided in the river, but does not signify consumptive use.

Table 9. 'Economic benefit' with financial compensations: the non-consumptive user is 'participatory'

User i	Priority	Unit benefit b_i (\$/1000 m^3)	Demand u_i (1000 m^3)	Volume provided q_i (1000 m^3)	Economic benefit b_iq_i (\$)	Cost of water pq_i (\$)	Financial compensation γ_i (\$)	Result (\$)
1	3	5	10	5	25	25	46	46
2	2	7	30	30	210	150	131	191
3	1	10	20	20	200	100	82	182
4 + 5	1	11	25	25	275	125	100	250
5	4	3	15	0	0	0	41	41
Total			$d = 100$	$Q = 80$	710	$pQ = 400$	400	710

Note: 4 + 5 symbolizes a coalition between users 4 and 5, in which the position on the list of priorities depends on the sum of the unit benefits of the two users.

user downstream who is best classified on the list of priorities. In the example there is no choice, because only user 5 is located downstream from user 4. The two together have a benefit equal to $b_4 + b_5 = \$11$ for each 1000 m^3 that reaches user 5, limited to 25 volume units. Thus, having formed the coalition, 'user 4 + 5' becomes the one with the highest priority. This means that the total consumption upstream from user 4 is limited to 80–25 = 55 volume units, while the cumulative demand for this subset of users is equal to 60 volume units. Therefore, 60–55 = 5 volume units must be rationed from some user located upstream from user 4. Among these, the worst placed on the priority list is user 1. The other 15 volume units can be rationed both upstream and downstream of user 4. The last on the list of priorities corresponds to the share of consumption of user 5 that exceeds the needs of the coalition 4 + 5. Table 9 shows the allocation of water in this situation, in which the run-of-the-river hydropower plant is 'participatory'.

Comparing Tables 8 and 9, it is observed that the total result in the basin, including user 4, rises from \$680 to \$710. Numerically, in this example, this difference is not very impressive. But in real cases this difference may be very significant. The 'gain' results only from the improved use of water, since each of the users 1, 2 and 3, individually, has a higher result in the case condensed by Table 9, than in the case condensed by Table 8. The result of coalition 4 + 5 must still be disaggregated, which naturally must be done proportionately to b_4 and b_5. In other words, the share of user 4 is $(8/11)\times 250 = 182$ and of user 5 is $(3/11)\times 250 = 68$. The result of user 5, considering the use of water in coalition with user 4 and individual use, is equal to 41 + 68 = 109. Therefore, all users are best served when the run-of-the-river hydropower plant decides to participate in the allocation, particularly itself, which in the example improves from a result of $160 to $182.

Generalization

Figure 4 shows the situation in which five users utilize water as an input to an economic process, including urban supply, from the same river or channel reach where there is a reservoir with a significant regulation capacity. As can be seen, user 3 takes in water at the lake shore, users 1 and 2 are upstream from the lake and users 4 and 5 are downstream.

In this example, the users are the same as in Figure 1. They fight for the same scarce resource that is the availability of water in the reservoir. Users 1 and 2 take out water that would have gone into the reservoir; user 3 takes water from the reservoir itself; users 4 and 5 can only be provided with water if the reservoir releases the amount of water needed. The greater the water use in the present time interval, the greater will be the probability of future rationing for the downstream users, due to reservoir emptying.

The water that is withdrawn from the reservoir for immediate use is more valuable when the reservoir is empty than when it is full. The expression for the value of water as a function of the stock of water in the reservoir can be derived by asking, for each level of stock, what the variation is of the gross domestic product of the river basin due to the addition of one unit of volume in the stock. In mathematical terms, this is the partial derivate of the function of basin production as a function of the stock of water. It can be demonstrated that the gross domestic product of the basin is maximized when price and water value are equal.

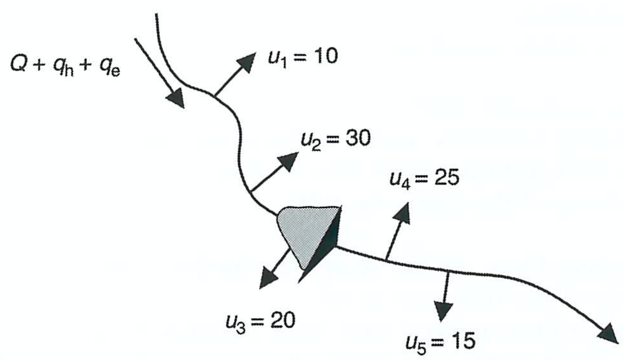

Figure 4. Five users in a river reach with a reservoir.

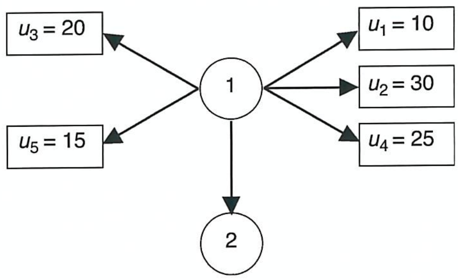

Figure 5. Representation by arcs and nodes in Figure 1.

The allocation of water among the users depends on the decision as to the variation of the stock of water in the reservoir. It also depends on knowing the water availability Q, which, in turn, depends on how the water available in the upstream reach of the river involved has been developed.

During this stage, it is appropriate that the equation (solution) be generalized to represent complex cases of river basins formed by many river reaches and with existing reservoirs. Since the authors are going to generalize, it is also appropriate to accept a case in which the streamflow taken up by any user will not be entirely consumed. The river basin is represented by a set of arcs and nodes. The arcs correspond to river reaches and the nodes to notable points, such as the confluence of rivers or reservoir sites.

Let it be agreed that each user i of water resources be characterized by four items of information:

(1) quantity of water u_i, which it is intended to taken in, during time unit Δt;
(2) consumption coefficient φ_i;
(3) intake node;
(4) restitution node.

For the example of Figure 1, the schematic representation would be made by only two nodes and one arc, as shown in Figure 5. All five users take in water at node 1 and have a consumption coefficient $\varphi = 1$. The restitution node is irrelevant in this case, since all of the water is consumed. On the other hand, for Figure 4 there would be three nodes, as represented in Figure 6. Node 2 represents the reservoir.

Each node k is characterized by:

- a set of nodes upstream, $M(k)$;
- a set of users that withdraw water at this node, $\Omega(k)$;
- a set of users that restitute water into it, $\Psi(k)$;
- the useful volume of the reservoir, $v_u(k)$

In the nodes where there are no reservoirs (nodes 1 and 3 of Figure 6), the 'working volume of the reservoir' is nil.

Figure 7 shows a more general case. User 1 wishes to take in $u_1 = 10$ volume units at node 3. Since water is available, the amount of water really taken in is $q_1 = 10$. This user returns 2 volume units at node 5. Therefore, his consumption

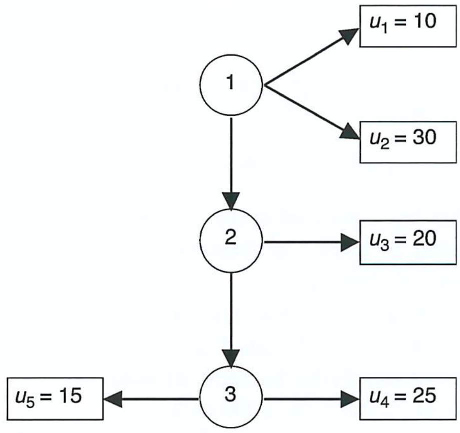

Figure 6. Representation by arcs and nodes in Figure 4.

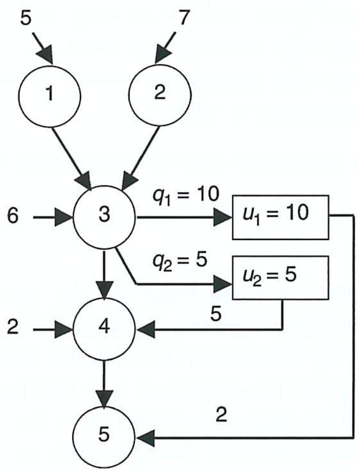

Figure 7. Representation of a general case with nodes and arcs.

coefficient is $\varphi_1 = (10–2/10) = 0.8$. On the other hand, user 2 wishes and is able to take in $u_2 = 5$ volumetric units at node 3. Since he returns the same 5 volume units at node 4, he is a non-consumptive user, typically a run-of-the river hydropower plant, with a consumption coefficient of $\varphi_2 = 0$.

In case node k has a reservoir, the water can be stocked or unstocked. Let $v_i(k)$ and $v_f(k)$ be respectively the initial and final volume of the reservoir, over interval Δt. The reservoir can be seen as a business which 'purchases' water at a low price when there is a lot on offer, during the wet period, to sell it for a high price when it is scarce, during the dry period. Let $p(.)$ be the unit purchasing or selling price of water for 'reservoir node k', which depends on the initial stock $v_i(k)$. Naturally, the greater is $v_i(k)$, the smaller will be $p[v_i(k)]$. At each Δt, the optimum allocation of water is obtained when the following linear programming problem is solved:

$$\text{Max}\ldots\sum_i b_i q_i + \sum_k [v_i(k) - v_f(k)] \times p[v_i(k)]$$

subject to

$$v_i(k) + a(k) + \sum_{j \in M(k)} e(j) + \sum_{i \in \Psi(k)} [(1 - \varphi_i)q_i] = v_f(k) + e(k) + \sum_{i \in \Omega(k)} q_i$$

for each node k, $e(k) \geq 0$, $v_u(k) \geq v_f(k) \geq 0$, $u_i \geq q_i$, where:

u_i = volume desired by user i at interval Δt;
q_i = volume allocated to user i at interval Δt;
b_i = net unit income ($/1000 m^3);
$v_i(k)$ = initial volume of reservoir located at node k;
$v_f(k)$ = final volume of reservoir located at node k;
$v_u(k)$ = useful volume of reservoir located at node k;
$p[v_i(k)]$ = price of water at node k ($/1000 m^3);
$a(k)$ = incremental volume contribution between node k and the nodes that are immediately upstream from it, during Δt;
$M(k)$ = set of nodes located immediately upstream from node k;
$e(j)$ = volume of outflow from node k during Δt;
$\Psi(k)$ = ensemble of users who restitute at node k;
$\Omega(k)$ = ensemble of users who withdraw water at node k;
φ_i = coefficient of utilization of user i.

Naturally, nodes without reservoirs are characterized by $v_i(k) = v_m(k) = v_f(k) = 0$.

Case Study

In Brazil, the largest reservoirs were built by the electricity sector in order to regulate the rivers to produce electric power. The national operator of the electricity system (Operador Nacional do Sistema Elétrico) (ONS) is the body responsible for dispatching the hydropower and thermal power plants, considering the electric power system interconnected on a national scale. The ONS uses computational models that determine the ensemble of operational decisions that are to fulfil energy demand in the most economical manner possible. The objective of these models is to minimize the expected value of the operational costs of thermal power plants and the costs resulting from energy rationing, over a planning horizon. The quick depletion of reservoirs for the purpose of increasing hydropower production and, consequently, diminishing the production of thermal power is not always advantageous, since it exposes the system to future rationing.

The simulation of the operation of the north–northeast electricity systems by means of a model similar to that used by the ONS enables the stored volume of any reservoir in these systems to be related to its value in water from the viewpoint of the electricity sector. The value of water is calculated in this model as the partial derivate of the objective function (thermal cost plus cost of rationing), with respect to the volume stored. In other words, the value of water in the reservoir, from the standpoint of the electricity sector, shows how much the cost would be reduced if an additional volume unit were available in the reservoir.

Figure 8 shows the variation in the value of water throughout the five-year period of 1999–2003, in the case of Sobradinho reservoir, on São Francisco River, considering 60 hydrologic scenarios taken from the historical series of

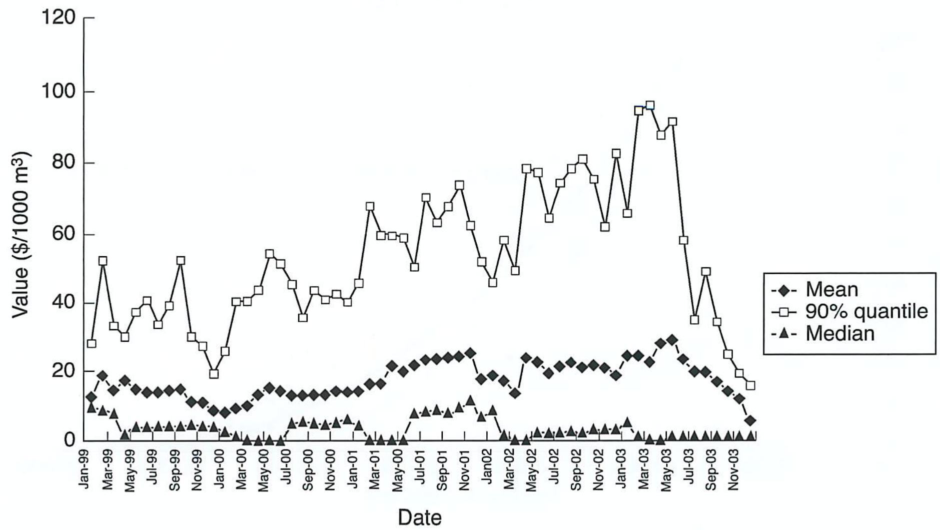

Figure 8. Evolution of the value of water at Sobradinho.

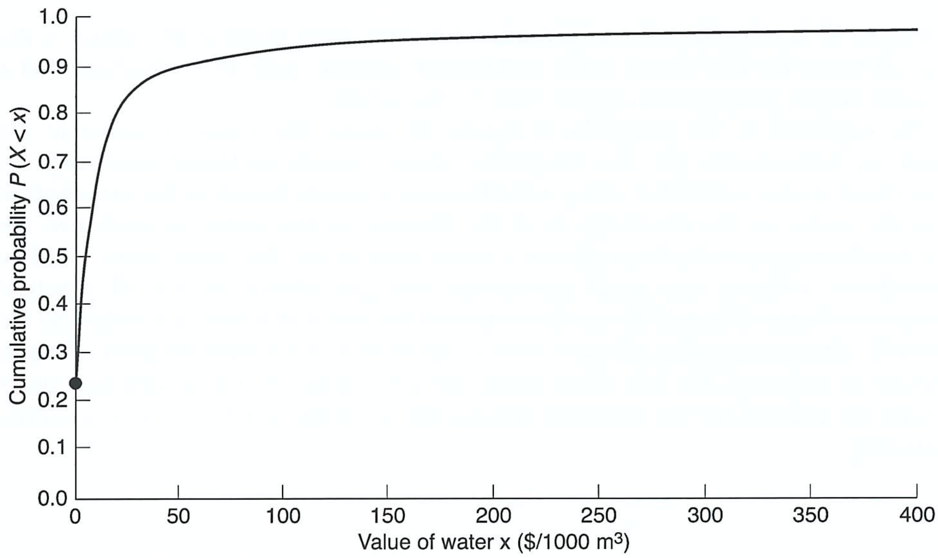

Figure 9. Cumulative distribution of water value at Sobradinho.

streamflows observed from 1931 to 1990. Since it would be impossible to visualize 60 curves, the choice was made to show only three curves that represent the median, mean and 90% quantile of the 60 observations. The high asymmetry should be observed: the median is in the order of $5/1000 m^3, whereas the 90% quantile fluctuates around $50/1000 m^3, almost reaching $100/1000 m^3, in April 2003.

Figure 9 shows the cumulative distribution of probabilities for the *X* value of water at Sobradinho. The probability that the value of water is nil (full reservoir) is 0.2, and that it is less than $50/1000 m^3 is approximately 0.9.

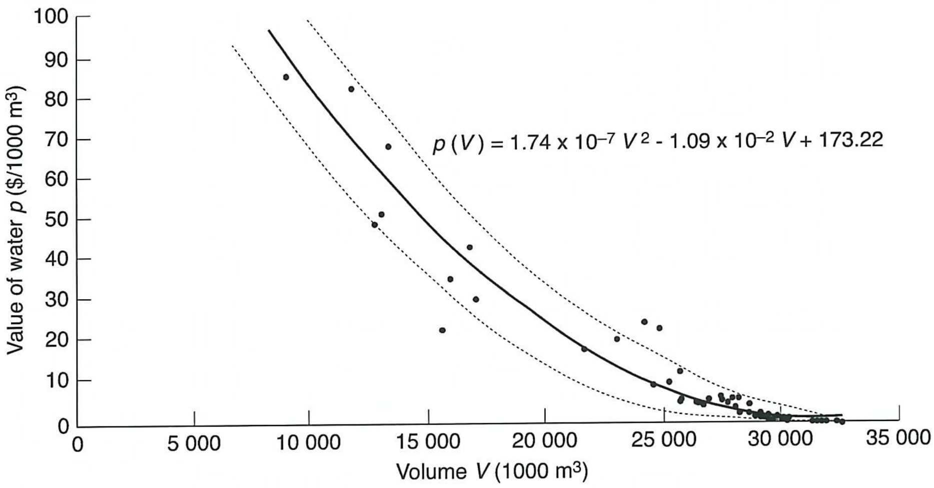

Figure 10. Value of water as a function of the volume stored at Sobradinho.

Figure 10 summarizes the information for 2000. Each point in the graph is the annual mean for the 'water value and stored volume' pair. It is observed that a second-degree polynomial adjusts well to the points.

The equation of the parabola in Figure 10 relates the value of water to the stock at Sobradinho for the electricity sector (north–northeast systems). As described, it was calculated using a mathematical model based on the assumption that the water in the reservoirs is at the disposal of the sector to minimize the electrical energy production costs on a nationwide scale. The other users could be considered without any great conceptual changes. However, for all practical purposes, Figure 10 could be used to establish the price of water. According to the 'benefit' rationing system, all users with a unit benefit lower than the price of water should be rationed. On the other hand, the price paid by the unrationed users would be utilized for the financial compensation of those who have to undergo rationing.

Notes

1. Human consumption should not be confused with urban consumption. Urban consumption includes a few items that have nothing to do with human consumption, such as watering gardens and washing cars and pavements.
2. *P*('event') means the probability that the 'event' will occur. In the specific case, the 'event' is the occurrence of an operational drought, characterized by smaller water availability than demand. When this probability is almost equal to zero, it is a situation in which water is plentiful. When it is significantly greater than zero, for instance greater than 0.05, it is a situation in which water is scarce, typical of a semi-arid region.
3. In the USA the most frequently used unit of volume is the acre-foot, which corresponds to the volume contained in a parallelepiped, with one acre as the base and one foot high. In this paper the authors adopt the unit of volume of 1000 m^3, which corresponds to a cube with a 10 m edge.
4. In Chile, the initial distribution of water use rights was auctioned. However, no distinction was made as to the hierarchy of access to water during periods of rationing. In other words, Chile implicitly adopted the 'linear' type of rationing system.

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