

INTERNATIONAL ASSOCIATION FOR HYDRAULIC RESEARCH

SYNTHETIC HYDROLOGY AND SPILLWAY DESIGN

(Subject B.d.)

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SYNOPSIS

Synthetic daily streamflow sequences are used for determining the relationship among the controlling variables in hydrologic dam-safety analysis : the spillway capacity, the maximum normal water level and the dam crest level. A simple stochastic model is described that is able to produce such sequences. The reservoir regulation equation proposed is convenient for re-evaluating the operation constraints of an existing dam. The methodology is applied to the Furnas Dam, in the Grande River, Brazil.

RÉSUMÉ

Les séquences synthétiques journalières de débit sont utilisées pour déterminer la relation entre les paramètres de contrôle dans l'analyse hydrologique de la sécurité du barrage : la capacité de l'évacuateur, le niveau normal maximum de l'eau et le niveau de crête. Un modèle stochastique simple est décrit comme pouvant produire les séquences en question. L'équation de régulation des réservoirs proposée est appropriée pour la re-évaluation de les restrictions d'opération d'un barrage existant. La méthodologie est appliquée au barrage de FURNAS, sur le Rio Grande, Brésil.

INTRODUCTION

The smaller is the discharge capacity of a dam spillway, the lower must be the maximum normal water level (MNWL) of the associated reservoir in order to ensure enough attenuation storage. This is necessary to avoid overtoppings caused by adverse inflow hydrographs. Although the cost of a spillway increases with its capacity, the best economical choice is not obvious because the benefits of a dam, measured for example by power production, also increase with the MNWL. There are spillway designs which are revised years after construction due to either the availability of new data or the use of a better calculation procedure. In these cases one may change the MNWL, resulting on different operation constraints or even conclude that the spillway is not safe enough. Therefore to obtain good designs or to get safe operation rules for existing reservoirs, one must be able to determine for each spillway choice its associated adverse inflow hydrograph and MNWL.

The adverse inflow hydrograph for large and important dams, also called project hydrograph, is usually determined either through flood frequency studies or through meteorological studies. The latter includes transposition and maximization of storm potential coupled with the use of rainfall-runoff transfer functions. Several criticisms have been raised against both approaches, such as:

a) the meteorological studies aim to calculate the inflow hydrograph upper limit based on the physical understanding of the water flux process in the earth-atmosphere environment. It is understandable that errors may be made on the representation of the process itself and on the estimation of the model parameters. One is never sure, with this method, how large the errors might be. In fact, "it is not uncommon for the first estimated probable maximum flood to be lower than the biggest observed flood for a site" (1).

b) the flood frequency studies are usually based on small records of flows, from which an also small sample of annual maxima is extracted. Probability distributions are then fitted to this sample and used to calculate quantiles located on the right tail, for example the "10000 years flow". Some advocate the use of a sample of flows exceeding a threshold level in order to overcome this sample size limitation. However it has been reported (2) that this procedure is not always more efficient than the maxima annual method.

The approach adopted in this paper is to model the historical flow record as a whole. In this way all of the available information is extracted from the data. This allows synthetic floods to be constructed which may exceed the largest observed flood due to "the joint occurrence of events which are not themselves remarkable but are together capable of producing a large flood" (3). It is briefly described a stochastic model for the generation of samples of daily streamflow. This model is used to generate thousands of synthetic daily streamflow sequences, the length of each one being equal to the number of days of the flood season. These sequences are then used to perform dam-safety analysis.

The methodology is applied to a dam built to house a power plant.

THE STOCHASTIC MODEL

In recent years, several researchers have stressed the require-

ment of parameter parsimony for any proposed stochastic hydrologic model. It is the opinion of the writers that the same parsimony principle should be applied to the assumptions adopted in the model building phase. In fact the two goals are conflicting, since a few parameters model can only be built on top of several assumptions, not always contemplated with empirical support. This is the case of the annual maxima frequency studies, when one is forced to adopt some probability distribution with few parameters due to the small number of events on the record. For instance, the use of type I Extreme Value Distribution (Gumbel), imposes the calculation of only two sample moments. Yet there are several assumptions underlying this method that can only be accepted if the corresponding daily stochastic process is of a very special type.

Fortunately there is plenty of information in the daily streamflow series that allows building models that resemble the data more properly. In this way, due consideration can be given, for example, to time-dependence and non-stationarity. On the other hand, the extreme value distribution turns out to be difficult to represent by a closed-form mathematical expression and the most convenient approach is the Monte Carlo method.

There are several stochastic models for daily streamflow proposed in the literature, for example (4), (5), (6) and (7). In general these models are based on varying degrees of theoretical assumptions about the process. The model described in the ensuing text might not represent the best balance of the parameters versus assumptions conflict. In fact it is biased towards minimizing the role of the assumptions in favour of empirical evidence.

Let $Q(t)$ be the mean flow on day t and

$$X(t) = Q(t) - Q(t-1) \quad (1)$$

$X(t)$ is classified in a three way table according to the following criteria:

A -	$X(t) \geq 0$	→	$a = 1$
	$X(t) < 0$	→	$a = 2$
B -	$q_{j-1} \leq q(t-1) < q_j$	→	$b = j$
C -	$\tau_{m-1} \leq t < \tau_m$	→	$c = m$

The vector $q = (q_0, q_1, q_2, \dots, q_j, \dots, q_r)$ partitions the range of daily flows into intervals whereas the vector $\tau = (\tau_0, \tau_1, \tau_2, \dots, \tau_m, \dots, \tau_s)$ partitions the flood season duration into s intervals. Therefore each value $X(t)$ may fall in one of the 2rd classes, according with the associated set (a, b, c) . The class marks should be selected according to the peculiarities of data. For example, one may guess that the falling (or rising) limb of the hydrographs behave differently for high and low flows and choose, by visual inspection, a component of q which will divide the two "states". Analogously one may observe that the floods in February "look different" from those of January and therefore choose the last day of January as one of the components of τ . Care must be taken to avoid classes with scarcity of sample points. In fact the number of observations in each class should be large enough to allow the use of the associated empirical distribution.

The persistence of daily streamflow is incorporated into the model through a seasonal two state Markov chain representation.

$$\pi_c = P(X(t) \geq 0 | X(t-1) \geq 0) \quad (2a)$$

and

$$\phi_c = P(X(t) < 0 | X(t-1) < 0) \quad (2b)$$

Where c depends on the t value, according to classification C .

Once the class mark vectors q and \bar{r} have been established, estimation of the transition probabilities $\pi_1, \phi_1, \pi_2, \phi_2, \dots, \pi_s, \phi_s$ and the grouping of the observed $x(t)$ values according to the corresponding (a,b,c) set, is a simple matter of data manipulation. Each year of synthetic daily flow is produced according to the following scheme:

- I) $t = 0$; sample $q(0)$ from the last-day-of-dry-season flow empirical probability distribution; $a = 1$
- II) $t = t+1$
- III) set value of b according to $q(t-1)$ and of c according to t
- IV) sample u value from uniform $(0,1)$ distribution
- V) if $a = 2$, go to (VII)
- VI) if $u > \pi_c$ then $a = 2$ and go to (VIII)
- VII) if $u > \phi_c$ then $a = 1$
- VIII) sample $x(t)$ value from the empirical distribution of the (a,b,c) class
- IX) $q(t) = q(t-1) + x(t)$
- X) if t is not the last day of the flood season go to (II)

DAM-SAFETY ANALYSIS

Usually a proposed spillway is tested through a routing calculation with the adverse inflow hydrographs, assuming the MNWL as the initial condition for the reservoir state. From these simulations one gets the maximum water level (MWL), on top of which it is added an allowance to wave run-up due to wind speed. These two levels are then compared with the dam crest level and account is given to possible hazards, considering the dam type. The eventual underdesign (or overdesign) should be corrected by changing either the crest level, or the MNWL or the spillway capacity.

Alternatively one may test the proposed spillway by calculating the required MNWL through a backward routing calculation with the adverse inflow hydrographs, assuming the MWL as a boundary condition for the reservoir state. This approach is particularly suitable when one has to re-evaluate the operation constraints of existing dams.

Following the latter alternative, it is necessary to calculate the minimum attenuation storage (which might be zero) sufficient to prevent overtopping the MWL. For an adverse inflow hydrograph i , the attenuation storage, $s(i)$, can be calculated as

$$s(i) = \max_t [s(t,i) = \max(0; s(t+1,i) + q(t,i) - d(s(t,i), s(t+1,i)))] \quad (3)$$

where

- t indicates the time, $t=h, h-1, \dots, 1$
- h is the last day of the flood season
- i indicates the adverse inflow hydrograph, $i=1, 2, \dots$
- $s(h,i) = 0, \forall i$
- $s(t,i) = 0$ implies that the water level is MWL
- $q(t,i)$ is the inflow to the reservoir on day t
- $d(s(t,i), s(t+1,i))$ is the outflow from the reservoir through the spillway on day t .

As the actual streamflow sequence is not known a priori, the attenuation storage S is to be considered as a random variable. Its probability distribution can be inferred from the set $\{s(i), i=1,2,\dots\}$, obtained from the use of equation (3) over thousands of synthetic sequences. The project attenuation storage is equal to the quantile s_α , defined as $P(S > s_\alpha) = \alpha$. The inverse of α is the selected recurrence time for the spillway, which for important dams usually varies between 1000 and 10000 years.

The MNWL is calculated entering on the storage-level relationship with the value of $(v_{\max} - s_\alpha)$ where v_{\max} is the storage of the reservoir at MWL.

CASE STUDY

The Furnas hydroelectric plant is situated in the southeastern region of Brazil, on the Grande River, the largest tributary of the Paraná River*. It lies approximately 320km to the north of São Paulo and 400 km northwest of Rio de Janeiro. Its basic purpose is power generation with its installed capacity of 1,216 MW. One of the more important characteristics of the Furnas project is the creation of a large storage reservoir whose objective is the long term regulation of the Grande River with consequent benefits to a series of hydroelectric schemes downstream. The catchment area of the Grande River upstream of the Furnas Dam is 52,000 km² and the mean discharge is about 900 m³/s. After enlargement and with a pool level at elevation 768.00m, the reservoir occupies an area of 1,440 km² and has an accumulated volume of 22.95 km³.

The final layout, determined by geological and topographical conditions, located the structure in the left abutment and a 127 m high zoned rockfill dam, arched in plan (crest length of 550m) over the riverbed and right abutment. Construction was started in 1958 and the first generating unit entered into service in 1963. The reservoir was designed for a MNWL of 766.50m and a MWL of 769.30m. The available head drop is about 95m. The spillway design, developed during the fifties, considered two constraints: (a) the spillway capacity at the MWL should be at least 13000m³/s, which would correspond to the largest Creager coefficient from southeast Brazil recorded at that time; (b) the routing of a particular project hydrograph, starting from the level MNWL should not overtop the MWL, even with one of the gates closed; this project hydrograph was defined based on regional frequency analysis of annual maxima and mean annual flow, for a recurrent interval of 10000 years (peak daily flow of 18000m³/s). As a result it was constructed a spillway controlled by 7 radial gates, 11.50m wide by 15.80m high, mounted on splitters 3.0m wide with a rounded front. The crest profile was determined by the flow lines for a head of water of 15.0m above the crest, located at the 750.80m. From the spillway crest the water is conveyed by a chute to a skijump and its energy dissipated in the river downstream.

Later consideration of other adverse inflow hydrographs have suggested that the spillway was overdesigned and that six gates would have been enough. In the seventies, it has been studied the raising of MNWL (9). In the following text the Furnas dam safety analysis is done in the light of the methodology proposed in this paper in order to illustrate its applicability.

* Technical data about Furnas Dam was extracted mostly from [8].

A 32 years record of natural daily streamflow (1930-31 to 1961-62) was used as input to the stochastic model. The class marks were chosen as $q_0 = 0$, $q_1 = 1000$, $q_2 = 2000$, $q_3 = \infty$ (m^3/s) and $\tau_0 = \text{Dec. 1}$, $\tau_1 = \text{Jan 1}$, $\tau_2 = \text{Feb 1}$, $\tau_3 = \text{Mar 1}$, $\tau_4 = \text{Apr 1}$ and $\tau_5 = \text{May 1}$.

100,000 years of 151 days long synthetic hydrographs were generated by a VAX 11/780 computer in 90 min of CPU time. The greatest generated daily flow was $12332 m^3/s$. Figure 1 shows a comparison between the sample accumulated distributions of maximum annual flow, obtained from the 32 years of historical record and from the 100,000 years of the synthetic sequence. Gumbel probability scale was used.

The good matching evident by eye inspection, can be confirmed by the chi-squared goodness-of-fit statistic of 1.01, using six grouping intervals. It is interesting to notice that the maximum flow in the 32 years of record has a return period of 61 years. It also can be seen that the estimated 10000 years flow is about $9500 m^3/s$ whereas the extrapolated Gumbel distribution yields a slightly greater value. In fact, in this case the synthetic streamflow approach and the use of Gumbel distribution fitted to the annual maxima are practically equivalent, as far as peak flow-return period relationship is concerned. The former method has the advantage of producing whole hydrographs, rather than just the peaks.

From the 100000 synthetic sequences only 28 were considered as "adverse hydrographs" for dam safety analysis. The adopted criteria was to select the hydrographs with peak flows greater than $9000 m^3/s$. The MNWL's calculated for each of these hydrographs through equation (3) were ordered allowing the estimation of the 10000 years project value (the 10th lowest level). Table 1 shows the results for 7, 6 and 5 gate openings. It can be seen that the

TABLE 1
DAM SAFETY ANALYSIS FOR FURNAS HYDROPLANT
(MWL = 769.30m)

NUMBER OF GATES	SPILLWAY CAPACITY AT MWL (m^3/s)	10000 YEARS MNWL (m)	LOWEST LEVEL (m)
7	13000	769.30	769.30
6	11143	769.30	767.75
5	9286	769.26	*

* NOTE. For 5 gates there were 3 synthetic sequences, out of 100000, which could not be accommodated even with the use of the total reservoir volume.

six gate solution suggested at the design phase is indeed quite reasonable.

The 10000 years MNWL is equal to the MWL even for only six gate openings, meaning that no attenuation storage is needed in the actual Furnas reservoir. Consequently the MNWL could be raised from 766.50m to 769.30m, which is equivalent to $4.05 km^3$ of gain in stored volume.

At least four other aspects have to be considered when modifying the MNWL : (a) change or adaptation of the spillway gates; (b) effect on the reservoir flooded area; (c) safety of the downstream dams; and (d) flood mitigation in downstream areas. A careful feasibility study (9)

has examined the possibility of raising the MNWL to 758.00, giving due consideration to the first two aspects. For this intermediate level rising, the gain in stored volume is 2.09 km³ which corresponds to a firm energy surplus of 689008 MWh/year. This calculation was done taking into consideration the other important generating plants, projected or under construction, of Southeast Brazil (total installed capacity of 34086 MW). This energy output can be produced by an "equivalent" hydroplant of about 130 MW. The last aspect (d) has also been considered in a different study (10) which came to the conclusion that a further 0,82 km³ attenuation storage is necessary to prevent, with a return period of 50 years, outflows from the dam greater than 4000 m³/s.

CONCLUSIONS

1. It is shown that the use of synthetic daily streamflow models can be a valid approach in the modeling of extreme flow - frequency relationship. The empirical support given by figure 1 demonstrates that convincing results can be obtained with a simple model, based on few assumptions.
2. Synthetic daily streamflow sequences are useful for determining the relationship among the controlling variables in hydrologic dam-safety analysis : the spillway capacity, the maximum normal water level (MNWL) and the crest level.
3. The suitability of adopting the backward routing, equation (3), in re-evaluations of operation constraints of existing dams is well demonstrated in the case-study of Furnas Dam.
4. Furnas spillway could have been constructed with only six gates. The safety of the existing dam allows the raising of the MNWL to the MWL, although this might not be economically feasible.

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TABLE OF FIGURES

Fig. 1. Synthetic flood frequency distribution and observed flows.
Distribution de fréquences des crues synthétiques et des débits observés.

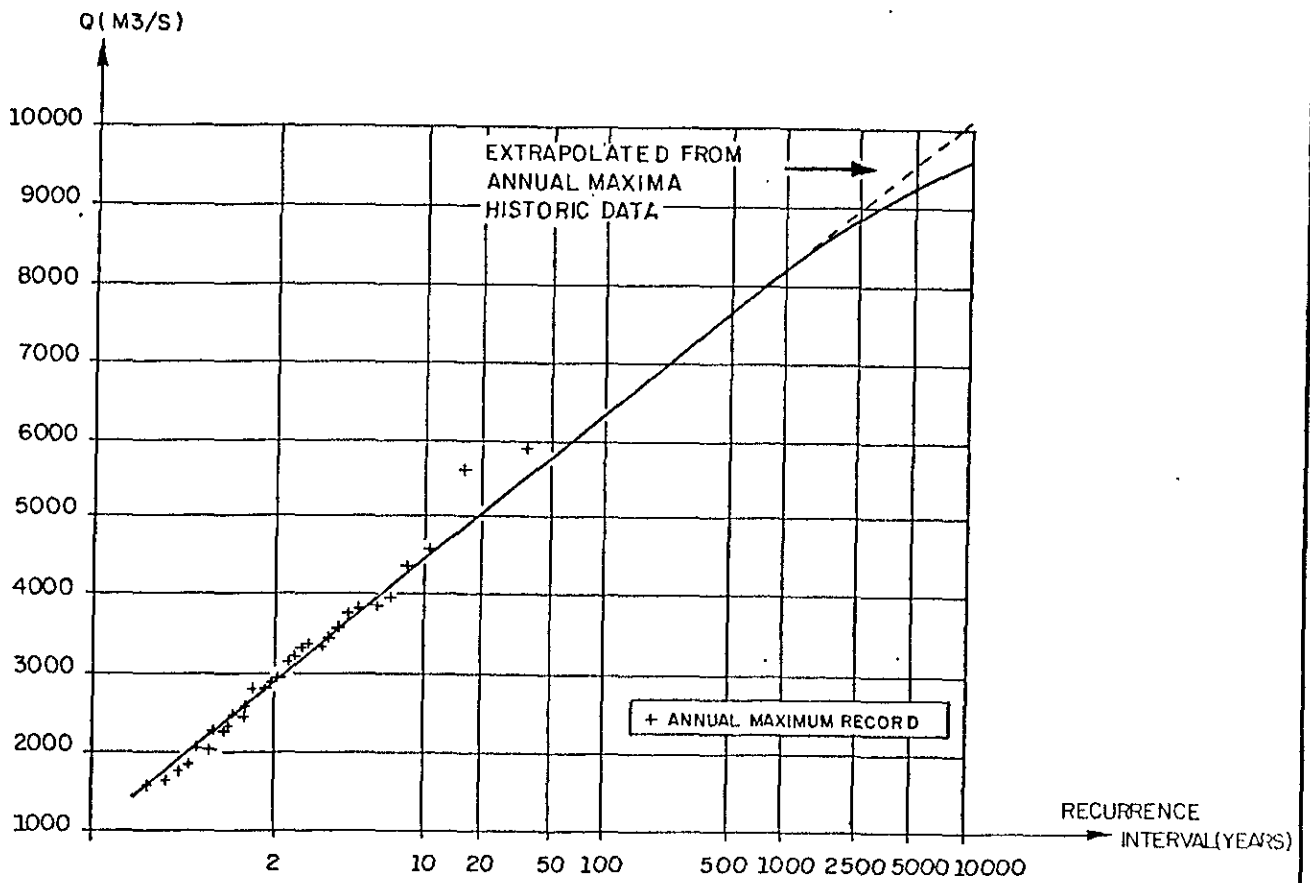


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